Developing, Delivering, and Researching Web-based Mathematics and Mathematics Education Courses

David A. Thomas
Professor of Mathematics Education
Department of Mathematics
University of Idaho, U.S.A
dthomas1@uidaho.edu

Cynthia S. Thomas
Research Faculty
College of Education
University of Idaho
cthomas@uidaho.edu

Abstract: The *Gateway to Mathematics Project* is developing and delivering web-based university level mathematics and mathematics education courses to dual enrollment high school students and K-12 mathematics teachers. This paper discusses our approach to content development and delivery, synchronous & asynchronous communications, automated practice and testing, collaborative problem-solving, and related research questions and methodologies. We also discuss the statewide partnership of schools, corporations, and agencies that is the foundation of this enterprise.

Introduction

In many rural Idaho communities, there are few if any opportunities for college-bound high school seniors or K-12 teachers to advance their mathematical education. Working with public and private sector partners, the *Gateway to Mathematics Project* (*GtM*) is researching, developing, delivering, and testing a sustainable solution to this problem (http://www.sci.uidaho.edu/Gateway). Our goal is to make high quality, post-secondary mathematics and mathematics education courses available to all Idaho residents. In the context of the *GtM*, the expression "high quality mathematics education" has academic, practical, theoretical, and pedagogical connotations:

- Academically, courses should reflect the values and standards of mathematics and mathematics education faculty throughout the Idaho university system and related educational, governmental, and industrial associations;
- Practically, courses should be easily transferable between university campuses and readily recognized by accrediting agencies (e.g., the Idaho Department of Education);
- Theoretically, courses should be systematically studied by researchers focused on the evaluation and improvement of web-based teaching and learning; and
- Pedagogically, courses should implement "best practices" identified by mathematics education researchers, rather than imitate traditional, face-to-face instruction.

GtM was funded by grants from the Fund for the Improvement of Education, US Department of Education and the Idaho State Board of Education. Dual enrollment students (i.e., high school students earning both high school and college credit) must apply through their high school, the project's partner institution. These partners are responsible for advertising upcoming courses, disseminating information, processing applications, facilitating registration, scheduling student access to course technologies, and project management/ coordination activities. Each high school partner designates one or more networked computers on their premises as *GtM* workstations, maintains course-related hardware and software installed on these machines, and supervises/assists students with administrative and/or technical questions. Local administrative technical liaisons act as points-of-contact for students, parents, and other interested persons. Unlike dual enrollment high school students, teachers may register for *GtM* graduate level mathematics and/or mathematics education courses as individuals without the involvement of a partner institution.

Course Management and Communication Technologies

All *GtM* courses use *WebCT* as the course management tool. *WebCT* provides password-protected access to course materials and communication technologies. A screen capture of the home page for one of the courses appears in Figure 1.

Figure 1: *WebCT* Course Interface

Students communicate asynchronously using *WebCT*'s threaded discussion system (See Figure 2). This technology is particularly valuable as students grapple with challenging problems designed to promote strategic thinking, problem-solving, thoughtful reflection, and mathematical communication. In this process, students exchange and test ideas in informal collaborations that, in some cases, become the bases for lasting relationships. Figure 2 shows an example of this sort of problem.

Subject: Defective Derivative

Message no. 4
Author: David Thomas (gcp)
Date: Friday, November 17, 2006 10:10am

```
You know from calculus that the derivative of x^2 (read "x squared") is 2x.  So, what's
wrong with the following argument?

1. Let x^2 = x + x + ... + x (x repeated x times)     (e.g., 4^2  =  4 + 4 + 4 + 4 = 16)
2. If you take the derivative of both sides you get (x^2)' = 1 + 1 + ... + 1 = x
3. So (x^2)' = x
```

Figure 2: *WebCT* Discussion Forum

Students meet in synchronous chat sessions timed to accommodate their individual schedules. In the case of dual enrollment high school students, chats occur during the regular school day and involve at most four students. These chats occur 2 – 3 times per week and last 20 – 30 minutes. In the case of K-8 teachers, most chats occur in the evening and often involve up to 10 participants. These chats occur once per week and last 30 – 50 minutes.

Synchronous chats are conducted in *Centra* (see Figure 3), a web-based conferencing tool that supports Voice-Over-IP audio, application sharing, hand-written mathematics, and a number of other communication technologies. In this setting, all participants may speak and be heard, hand-write mathematics, annotate figures, type in text, and share applications running on the instructor's computer. This last capability makes possible technology training (e.g., *The Geometers Sketchpad*). Together, the asynchronous and synchronous communication technologies provide an environment for rich mathematical and pedagogical dialogue.

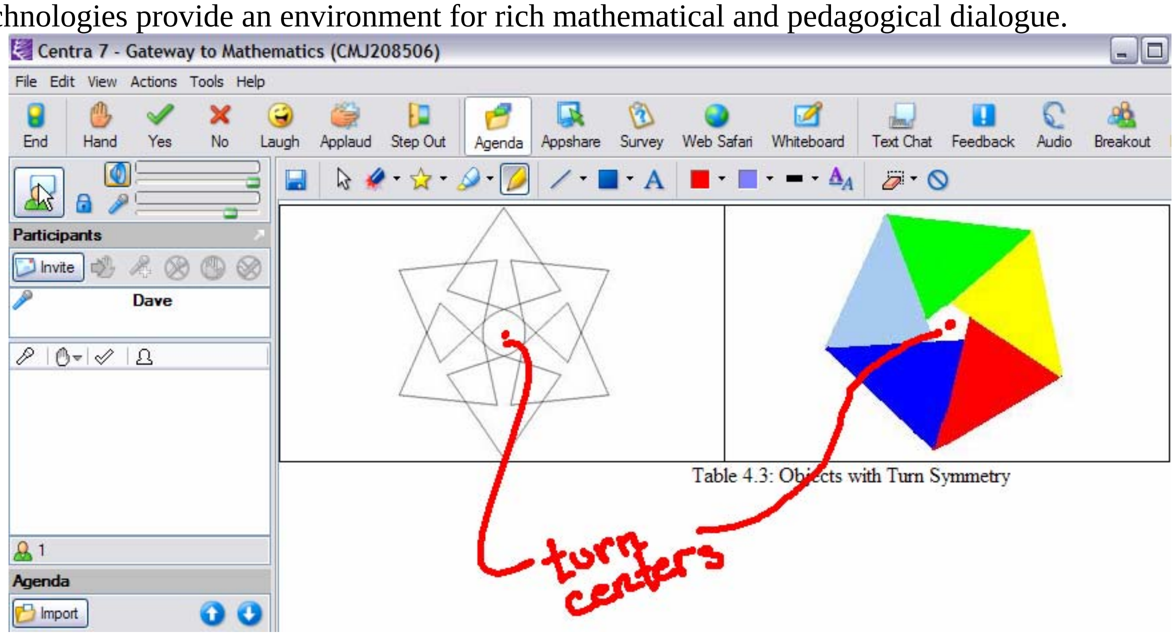


Figure 3: *Centra* Interface

Math 170 Analytic Geometry & Calculus I

Math 170 is a 4 semester credit course offered to dual enrollment high school students interested in mathematics, science, and engineering. Based on the university's traditional introductory calculus course and *WebCalc* (Allen, Stecher, Yasskin, 2001) developed at Texas A&M University, Math 170 presents a traditional set of topics (See Figure 4) using non-traditional pedagogies enabled by powerful communication and modeling technologies.

Table of Contents, Calculus I

R1. Basics
R2. Functions and Graphs
R3. Manipulating Functions and Graphs
R4. Review of Trigonometry
1. Qualitative Limits and Continuity
2. The Derivative
3. Formulas for Derivatives
4. Notation for Derivatives
5. Derivatives of Trigonometric Functions
6. Chain Rule
7. Applications of the Derivative
8. Higher Order Derivatives
9. Exponential Functions
10. Inverse Functions
11. Logarithm Functions
12. Applications of the Exponential and Logarithm to Economics
13. Mean Value Theorem
14. Newton's Method
15. Introduction to Curve Sketching
16. Concavity
17. Asymptotes
18. Optimization
19. Antiderivatives
20. Introduction to the Integral
21. The Fundamental Theorem of Calculus
22. The Method of Substitution
23. Applications: Area and Average Value
24. Applications: Volumes
25. Applications: Work

Figure 4: Math 170 Topics

The course "textbook" is published online using *Scientific Notebook* (MacKichan, 2005) (See Figure 5). Concepts, procedures, examples, and informal practice are written in a textbook-like style and displayed in webpage format. A Daily Schedule webpage provide access to informal practice, videos, Java applets, and other WWW-based information resources. Additional links provide access to *Bernoulli*, algorithmically generated and scored practice and quizzes aligned closely with exercise sets published in the online textbook (Link-Systems, 2005) (See Figure 6). In practice mode, *Bernoulli* provides several coaching and feedback options, including:

Show Me	Contains a built-in, step-by-step solution procedure.
Step by Step	Includes the ability to break the step-by-step solution into segments, some segments prompting the student to enter part/all of the next step.
Hint	Can be either the first step of the step-by-step solution or a static example.

Bernoulli automatically sends an email report to the instructor every time a student completes a set of practice problems. This report includes the actual items, student responses, and summative performance information. This information is useful in synchronous chats when students ask questions on *Bernoulli* practice items.

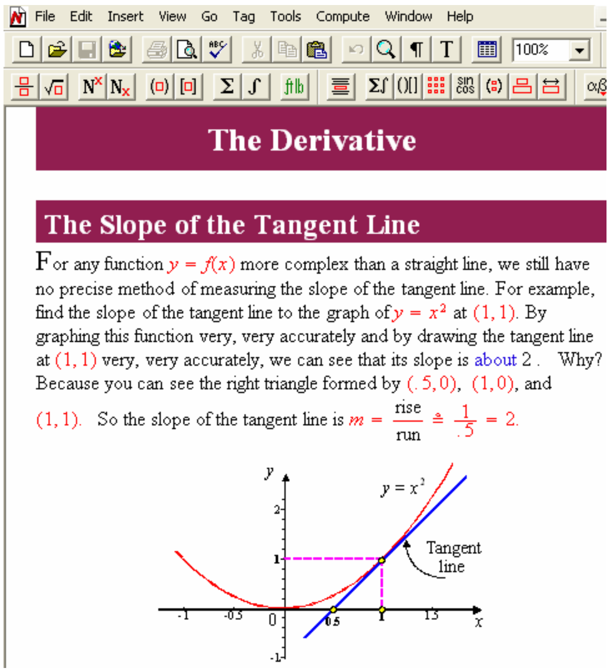


Figure 5: *Scientific Notebook*

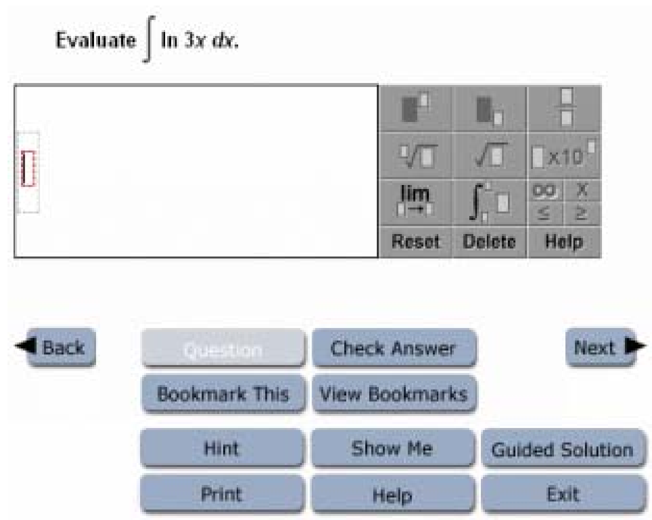


Figure 6: *Bernoulli*

Math 505 Number & Operations for Teachers

Many teachers have little knowledge of or experience with non-traditional concept models and algorithms for the arithmetic operations that form the core of the mathematics curriculum. In effect, their mathematical and pedagogical knowledge are limited to the methods used to teach them when they were students. This course is designed to provide teachers with alternative methods for thinking about numbers and performing arithmetic operations (See Figure 7).

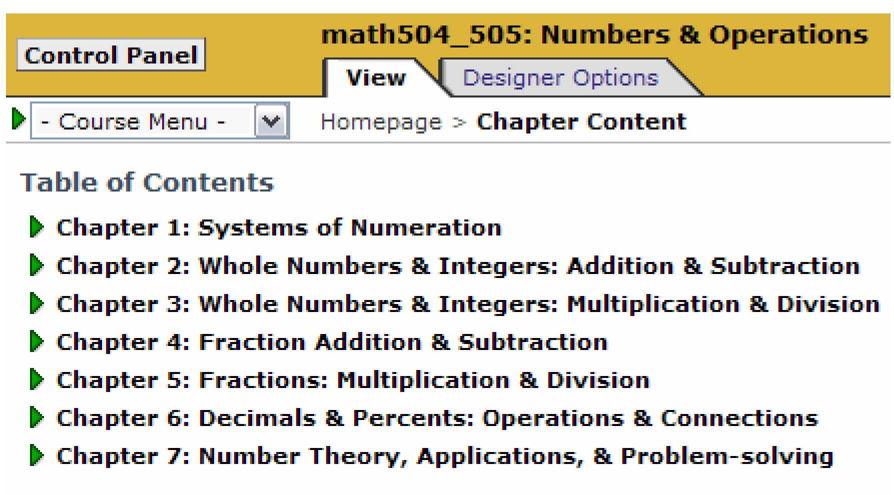


Figure 7: Number & Operations Table of Contents

The course emphasizes the use of multiple representations, expanded and alternative algorithms and technology-based modeling tools (See Table 1). Asynchronous and synchronous communication are conducted using *WebCT* threaded discussions and *Centra*.

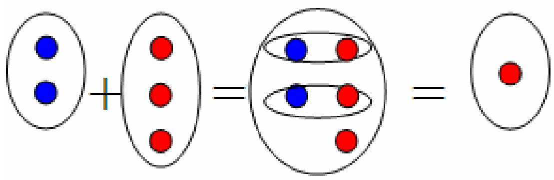
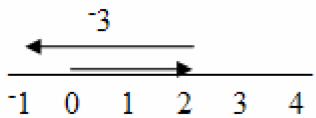
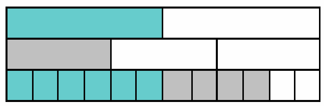
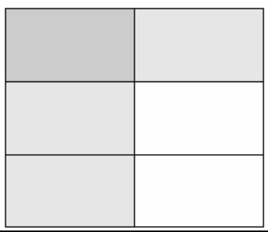
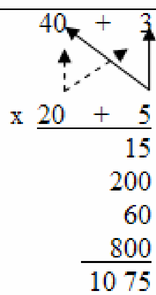
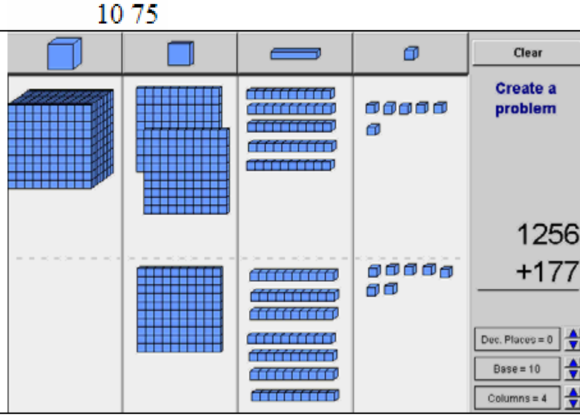
Operation	Concept Models	Interpretation
$2 + {}^{-}3 = {}^{-}1$	  	Each blue chip has a value of $^{+}1$. Each red chip has a value of $^{-}1$. 1 blue chip + 1 red chip = 0 Two vectors are arranged end-to-end representing the two terms in the expression.
$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$	 $\frac{1}{2}+\frac{1}{3}=\frac{6}{12}+\frac{4}{12}=\frac{10}{12}$ 	In the fraction bar model, the concept "common denominator" is modeled as a common divisor, in this case 1/12. In the area model, a common divisor is automatically generated by folding a unit square in half vertically and into thirds horizontally.
$8\overline{)143}$	10 + 7 + 0 8) 100 + 40 + 3 ⇒ 17 R7 -80 ↓ ↓ 20 → 60 ↓ -56 ↓ 4 → 7 -0 7	In this alternative algorithm, division is modeled as repeated subtraction. Multiples of the divisor are subtracted in quantities that are convenient to the student.
43 x 25	 	In this alternative algorithm, the factors are separated in binomials highlighting the place-value of each digit in the factors. Partial products are then formed, without using the "shift" rule or hiding the value of any partial product.
1256 + 177	 	*Base Blocks Addition* applet at the *National Library of Virtual Manipulatives* URL nlvm.usu.edu/en/nav/vlibrary.html

Table 1: Multiple Representations

Math 505 Geometry & Measurement for Teachers

The same traditional mathematical education that limited teachers' understanding of number and operations limited their mathematical and pedagogical knowledge of geometry and measurement. This course is designed to provide teachers with alternative methods for thinking critical concepts and procedures for teaching these subjects. In this course, measurement is interwoven with geometry, rather than taught as a separate topic (See Figure 8).

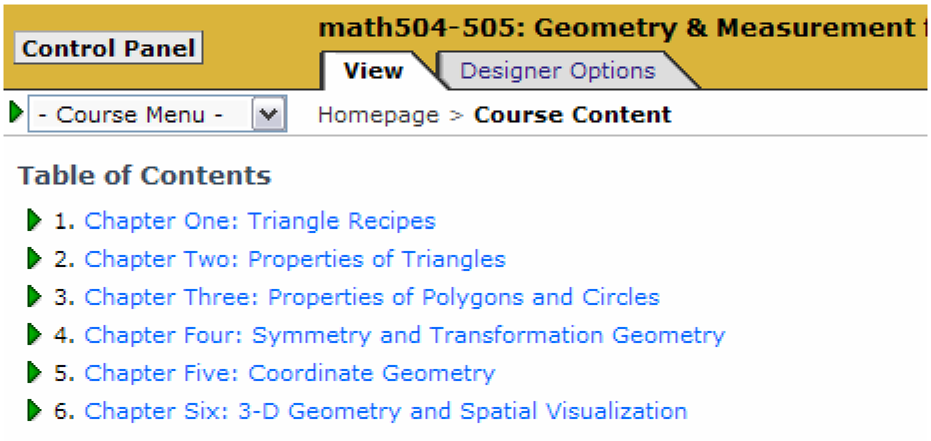
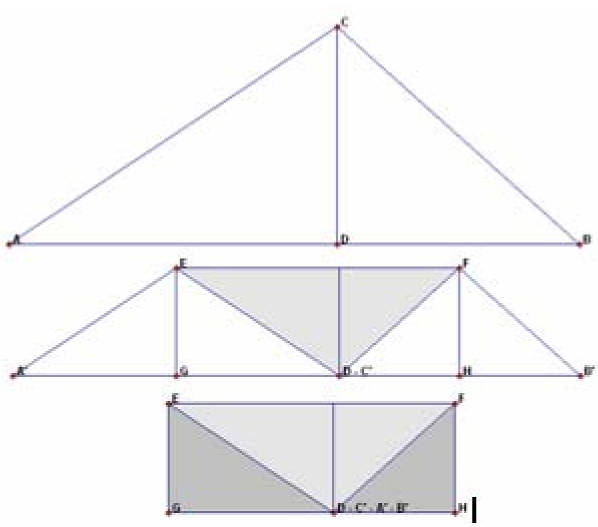


Figure 8: Geometry & Measurement Table of Contents

The course also emphasizes the use of multiple representations, expanded and alternative algorithms and technology-based modeling tools (See Table 2). Asynchronous and synchronous communication are conducted using *WebCT* threaded discussions and *Centra*.

Concept Models	Interpretation
	Paper-folding demonstration that the sum of the angles of a triangle is 180°.

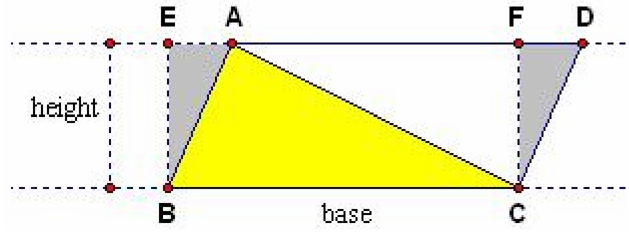
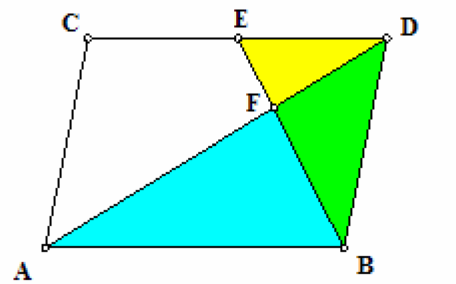
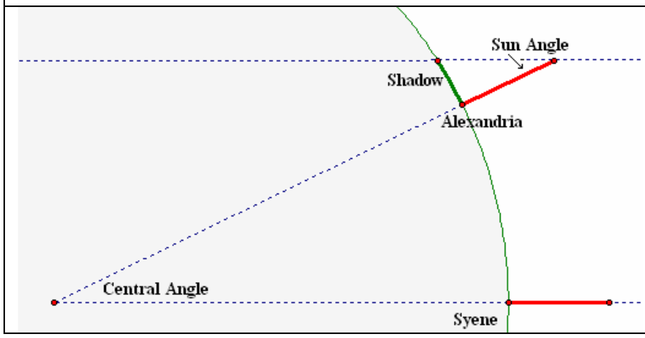
<table>
<tr><td>
</td><td>Geometers Sketchpad model demonstrating that the area of a triangle is half that of a rectangle with the same base and height.</td></tr>
<tr><td>
</td><td>Use The Geometers Sketchpad to investigate invariant features of all parallelograms constructed as shown, with E the midpoint of CD.</td></tr>
<tr><td>
</td><td>Use The Geometers Sketchpad to explain the logic used by Eratosthenes to compute the circumference of the Earth.</td></tr>
</table>

Table 2: Multiple Representations

Research Opportunities & Methodologies

In the last five years, significant progress has been made in the development of research methodologies for investigating teaching, learning, and collaboration in Asynchronous Learning Networks (ALNs). We now have persuasive evidence that online learning in Asynchronous Learning Networks (ALN) is equivalent to or superior to face-to-face instruction (The Sloan Consortium, 2005). An ALN is a teaching and learning environment designed for anytime/anyplace use. For instance, many ALNs use on-demand technologies (e.g., web pages, *Flash* movies, and java applets) to present information and messaging systems (e.g., email, threaded discussions, and journals) to facilitate student-student and student-teacher interaction. Typically, research data are analyzed using both statistical methods and the methods of Social Network Analysis (SNA) (Wasserman, Faust, Iacobucci, & Granovetter, 1994), an application of mathematical graph theory. In this context, the value of statistical analysis lies in its capacity to identify and measure the strength of associations and treatment effects; and the value of social network analysis lies in its capacity to explain how student-student and student-teacher communication produced those associations and effects.

Unlike communication in ALNs, students in Synchronous Learning Networks (SLNs) engage in real time dialogues using the use of chat room, whiteboard, and application sharing technologies. As one might expect, ALN and SLN learning environments have very different rhythms and purposes. In our experience (Beaudrie & Thomas, 2000), synchronous chats are particularly useful in small group discussions when setting goals, clarifying issues, negotiating meaning, resolving issues, and evaluating group progress. Alternatively, asynchronous messaging (with attachments) is the mechanism by which the detailed insights/contributions of individual students are shared, evaluated, revised, and ultimately incorporated in the development of group products (e.g., team reports). The *GtM* uses both statistical and SNA research methodologies in the study of teaching and learning in its web-based courses. In so doing, we hope to devise and validate procedures suitable for use in both ALN and SLN learning environments.

Background Research

Spring term of 2005, David Thomas taught three web-assisted undergraduate mathematics courses at the University of Idaho: Math 235 Mathematics for Elementary Teachers I (i.e., Number & Operations) ; Math 236 Mathematics for Elementary Teachers II (i.e., Geometry & Measurement); and Math 391 Modern Geometry. Course content materials developed specifically to meet the needs of preservice teachers were used. In each course, students met face-to-face with one another and the instructor three or four times per week, depending on the course. In addition, students engaged in a sustained, purposeful (e.g., required and rewarded) asynchronous dialogue focused on homework assignments. In order to promote the development of this sort of dialogue, students in each course were divided into four groups the first day of class based on their seating choices. For the rest of the semester, group members worked together at tables each class period and communicated outside of class in online threaded discussions.

During the semester, each group of students created, edited, and submitted several lengthy (graded) homework reports. As each homework report came due, students also prepared for and completed individual hour-long paper-and-pencil examinations covering the most significant concepts and skills addressed in the course. Their achievement as individuals was assessed via these examinations and term papers. Overall grading schemes were consistent in all three courses. The evidence of their engagement in online discussions was stored automatically on the university server hosting the online discussions. This dialogue, in the form of threaded discussions, was stored as long strings of textual data. A parser was written for converting this data into a useable file format for analysis. These data were entered into response matrix illustrated using the fictional data seen in Table 3. In this table, cell *(i,j)* indicates how many times the *i*-th (row) learner responded to postings by the *j*-th (column) learner during asynchronous communications. Notice that this matrix is not symmetrical. That is, not all learners engage one another in the same ways.

	John	Thomas	Anna	James	Peter	Mary
John	0.0	0.0	0.0	0.0	0.0	0.0
Thomas	0.0	0.0	0.0	0.0	0.0	0.0
Anna	3.0	0.0	0.0	0.0	0.0	0.0
James	2.0	0.0	0.0	0.0	0.0	0.0
Peter	0.0	0.0	0.0	0.0	0.0	0.0
Mary	0.0	0.0	0.0	0.0	0.0	0.0

Table 3: Discussion Forum Relation Matrix (Fictional Data)

Over 1500 messages from all three courses were combined into a single set representing 49 students and analyzed using *NetMiner*™ (Cryam, 2006). A graphical representation of the network structures that emerged during the semester is seen in Figure 9. In this figure, male students are represented by small icons and female students by large icons. Lines between students indicate messaging between students in the form of replies (i.e., not "broadcast" postings). Clearly, some students never engaged in this sort of messaging. This should not be interpreted to mean that these students never read messages or never spoke to group members in class. For now, that behavior remains unexamined. What is clear is that different groups developed different network structures. Graph theoretic and statistical analyses of these data yielded the following findings:

- Student asynchronous communication was conducted primarily within groups rather than between groups;
- Achievement on individual examinations was independent of gender;
- Approximately 55% of the variability in individual examination scores may be explained by the structure of online discourse.

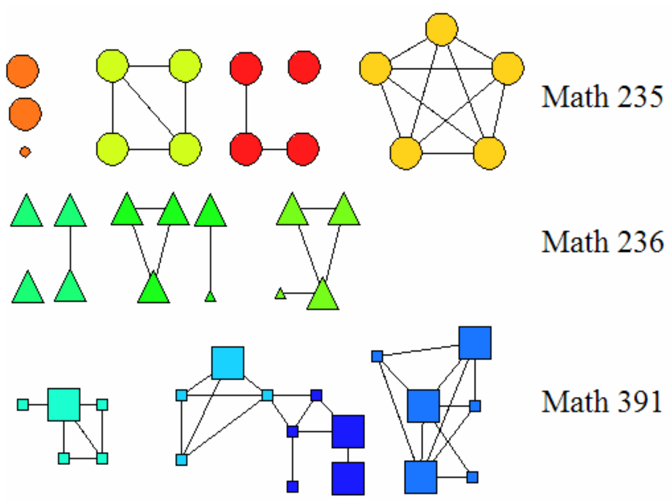


Figure 9: Communication Structure Visualization

These findings suggest the existence of underlying mechanisms that promote learning through student participation in asynchronous communications. Understanding those mechanisms will necessarily involve content analysis as well as improved measures on student nodal characteristics and emergent network structures. The fruits of this research could include better insights into "what works" in web-assisted and distance learning courses and how periodic informal analyses could benefit instructors and students while courses are still in progress.

References

Allen, D., Stecher, M., & Yasskin, P. (2001). WebCalc. The WebMath Group. Texas A&M University. Available on-line at http://www.math.tamu.edu/~webcalc/webcalc.html.

Beaudrie, B. and Thomas, D. (2000). Communication between problem solving groups in an on-line geometry course: Results and lessons learned. *Proceedings of MSET 2000 International conference on mathematics/science education & technology*. San Diego, California, February 5-8, 2000, 65-70.

Centra (2007). Synchronous communication technology. Available online at http://www.saba.com/products/centra/index.htm

Cryam (2006). *NetMiner II, Ver. 2.6.0,* Seoul: Cryam Co., Ltd. http://www.netminer.com

Link-Systems International, Inc. (2005). Bernoulli. Available on-line at http://www.link-systems.com/ PsT89D9CiqIAAC7a7W0/
MacKichan (2005). Scientific Notebook. Available on-line at http://www.mackichan.com/
The Sloan Consortium (2005). Journal of Asynchronous Learning Networks. Available online at http://www.sloan-c.org /publications/jaln/index.asp.
Wasserman, S., Faust, K., Iacobucci, D., & Granovetter, M. (1994)**.** Social Network Analysis: Methods and Applications. Cambridge: Cambridge University Press.
WebCT (2005). WebCT. Available on-line at http://www.wacom.com/index2.cfm